A front evolution problem for the multidimensional East model

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Based on joint work with: Fabio Martinelli (Roma Tre)

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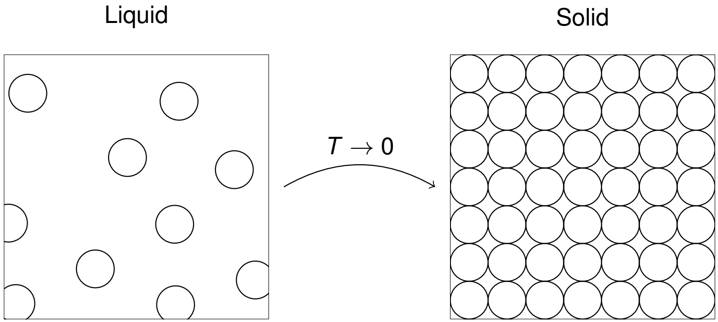
University of Geneva

Plan

- Multidimensional East model
- Front evolution problem
- Equilibrium behind the front
- Mixing time

Motivation



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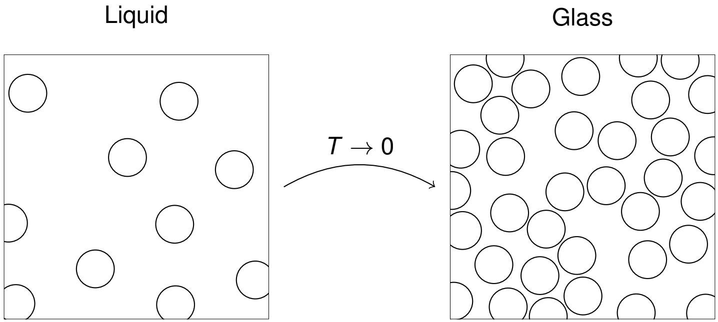

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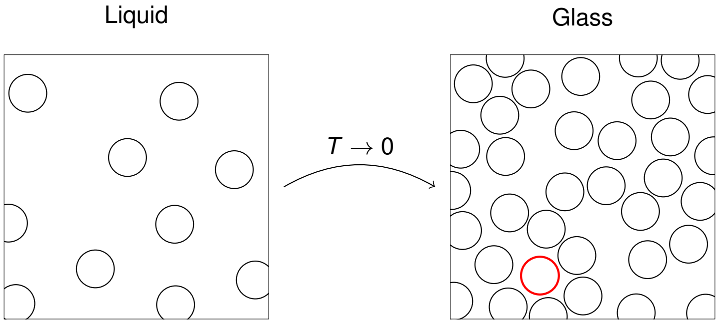

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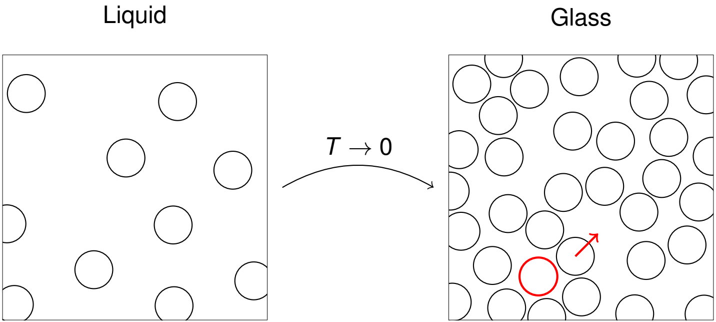

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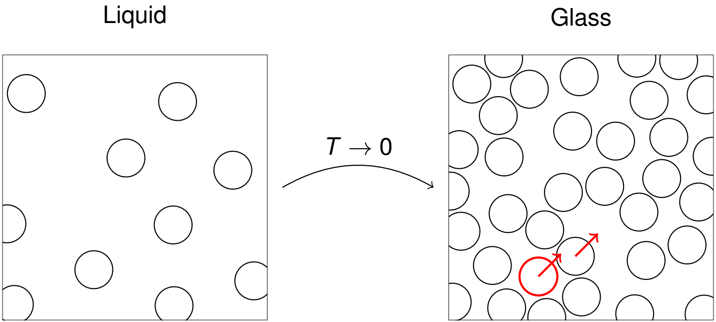

Multidimensional East model

- Markov process on $\mathbb{Z}^d$, parameter $q \in (0, 1)$.
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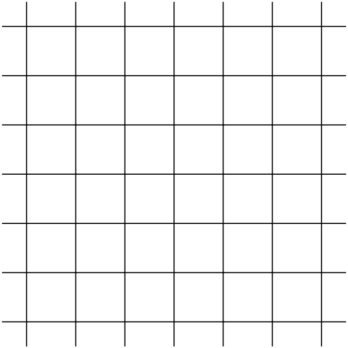

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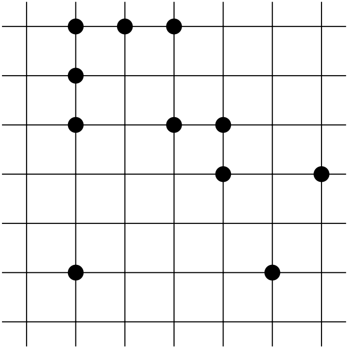

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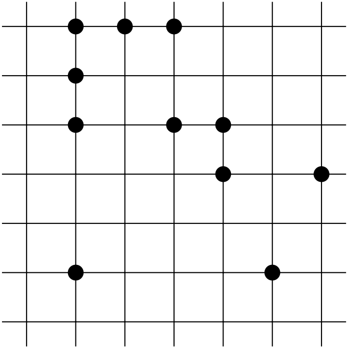

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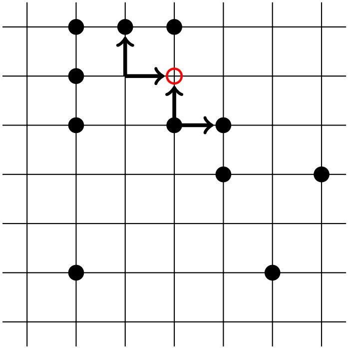

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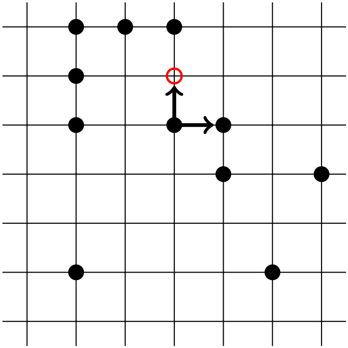

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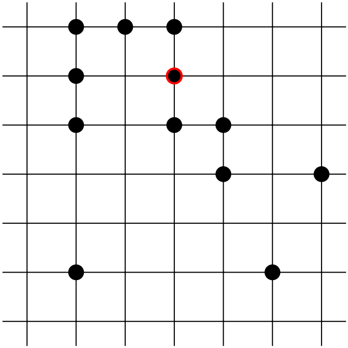

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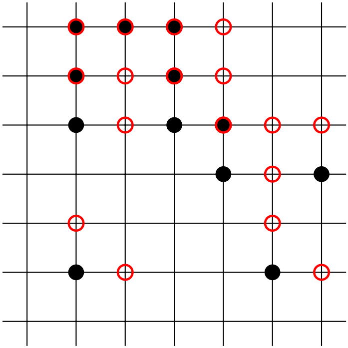
- $\mu = \bigotimes_{x \in \mathbb{Z}^d} \mu_x$ reversible.



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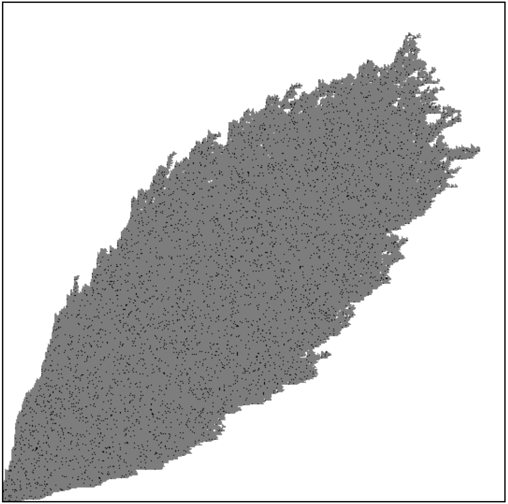
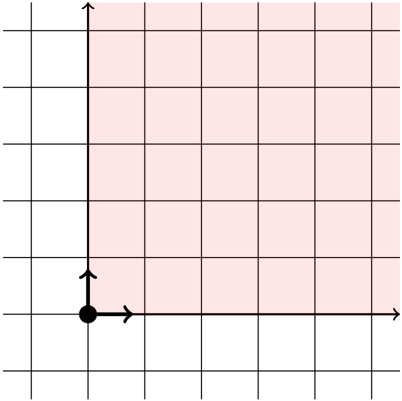
Simulation results



$\bullet$ = previously $\bullet$.

Front evolution problem

- Start with state ω_* with single vacancy at origin.
- $\bullet$ only on first quadrant.



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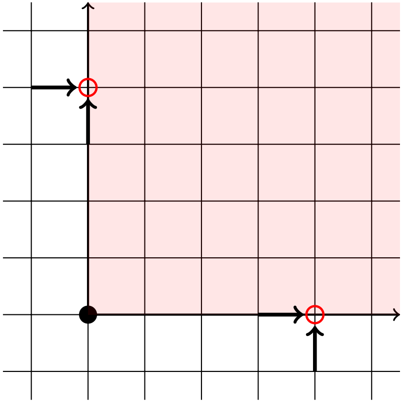

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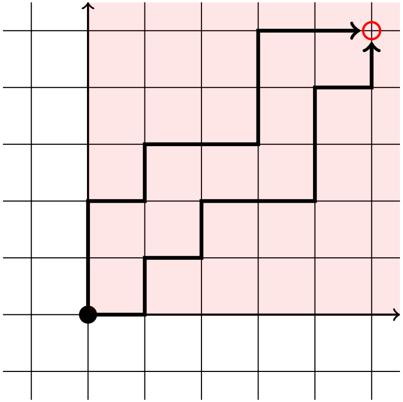
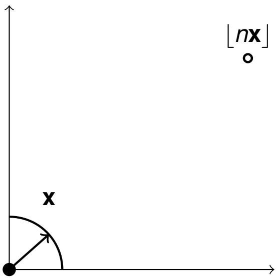
- Faster propagation to vertices away from axis.



Question: Is there a front velocity?

τ_x = infection time of $\lfloor x \rfloor \in \mathbb{Z}_+^d$, $\quad$ $\mathbf{x}$ = unit vector in $\mathbb{R}_+^d$.

$$\frac{1}{v_{\max}(\mathbf{x})} := \liminf_{n\to\infty} \frac{\mathbb{E}_{\omega_*}(\tau_{n\mathbf{x}})}{n}, \qquad \frac{1}{v_{\min}(\mathbf{x})} := \limsup_{n\to\infty} \frac{\mathbb{E}_{\omega_*}(\tau_{n\mathbf{x}})}{n}$$



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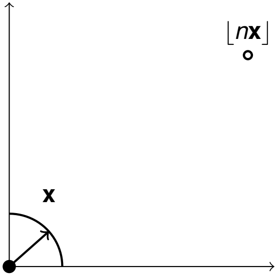

Main problems

Bounds on $v_{\min}(\mathbf{x})$, $v_{\max}(\mathbf{x})$.

Harder: Identify $\mathbf{x}$ for which $v_{\min}(\mathbf{x}) = v_{\max}(\mathbf{x})$.

Question: Is there a front velocity?

Theorem (O. Blondel '13)

In $d = 1$ *there exists a* $v = v(q)$ *such that* $v = v_{\min}(\mathbf{e}_1) = v_{\max}(\mathbf{e}_1)$ *for any* q*.*

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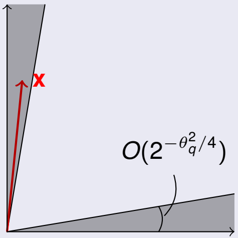
No bounds on $v_{\min}(\mathbf{x})$, $v_{\max}(\mathbf{x})$ for $d \geq 2$, $\mathbf{x} \neq \mathbf{e}$.

Small q behaviour of $v_{\max}(\mathbf{x})$, $v_{\min}(\mathbf{x})$

Write $\theta_q = \log_2(1/q)$. By (P. Chleboun, A. Faggionato, F. Martinelli '16) the spectral gap $\gamma_d(q)$ of the East model on $\mathbb{Z}^d$ is $2^{-\frac{\theta_q^2}{2d}(1+o(1))}$.

Theorem (Y.C., F. Martinelli '22)



If $d = 2$, $\mathbf{x}$ as in figure, then

$$
\begin{aligned}
v_{\max}(\mathbf{x}) &= v_{\min}(\mathbf{x})^{1+o(1)} \\
&= 2^{-\frac{\theta_q^2}{2}(1+o(1))} \\
&= \gamma_1(q)^{1+o(1)}, \quad q \ll 1.
\end{aligned}
$$

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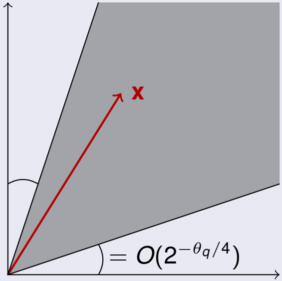

If $d \geq 2$, $\mathbf{x}$ as in figure $\Rightarrow \exists \alpha < 1$ s.t.

$$v_{\min}(\mathbf{x}) \geq 2^{-\frac{\theta_q^2}{2}\alpha} \gg v(\mathbf{e}), \quad q \ll 1.$$

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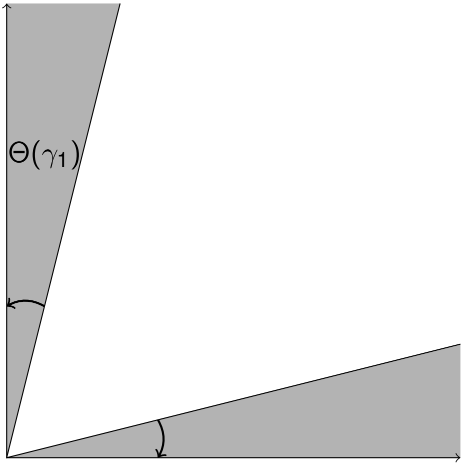
Theorem (Y.C., F. Martinelli '22)

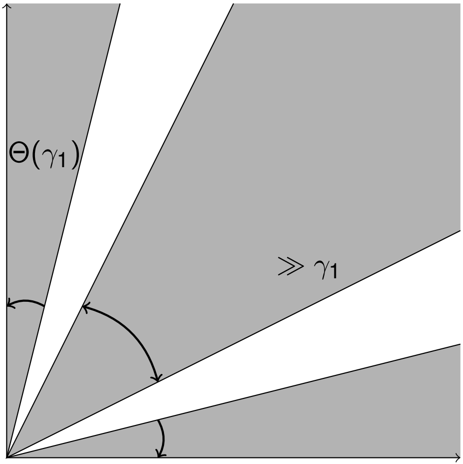
If $d \geq 2$, $\mathbf{x} \in \mathbb{R}_+^d$ s.t. $\min_i \mathbf{x}_i > 0$. Then

$$v_{\max}(\mathbf{x}) = v_{\min}(\mathbf{x})^{1+o(1)} = 2^{-\frac{\theta_q^2}{2d}(1+o(1))} = \gamma_d^{1+o(1)}(q), \quad q \ll 1.$$



$$q \to 0, \qquad \gamma_d \to 0$$

$$T \to 0, \qquad T_{\text{rel}} \to \infty$$



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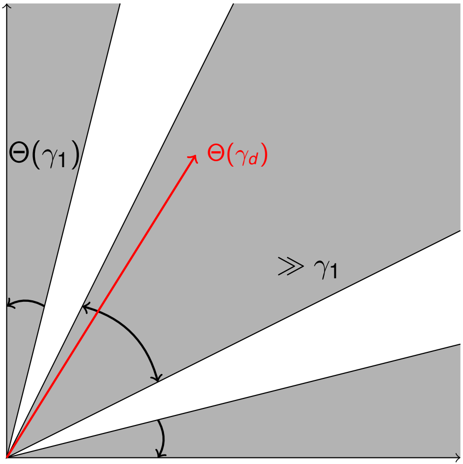

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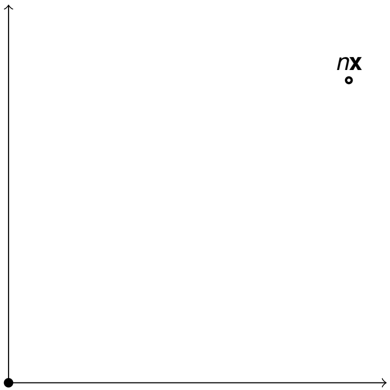
Main ingredients for $v_{\max}(\mathbf{x}) \le 2^{-\frac{\theta_q^2}{2d}(1+o(1))}$ as $q \to 0$



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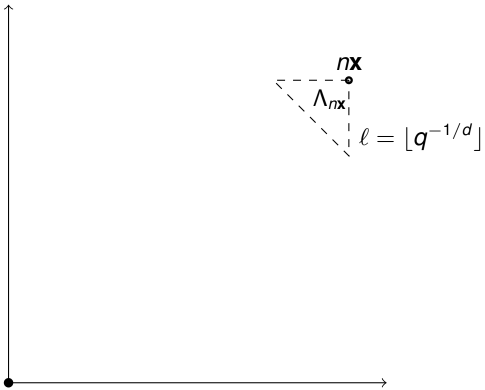

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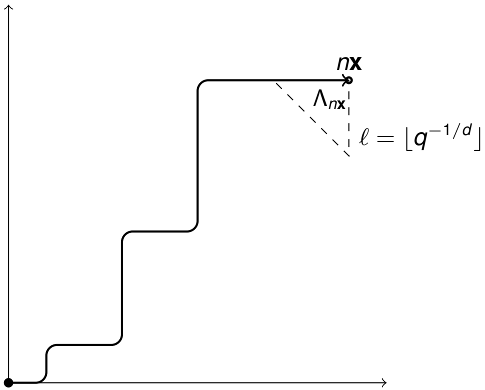

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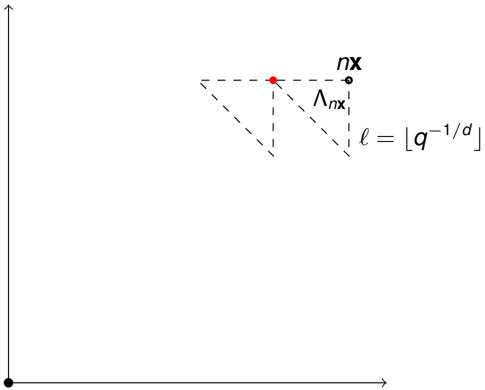

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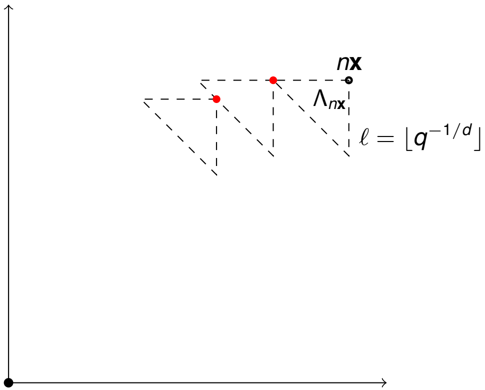

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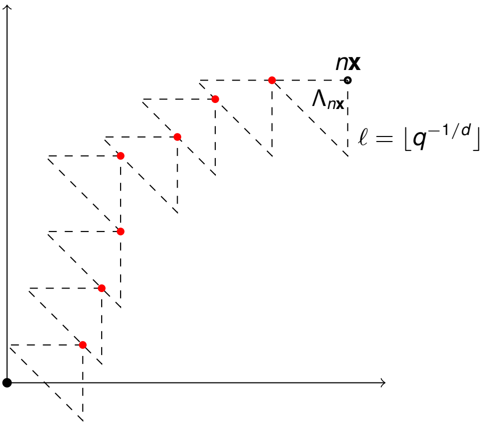
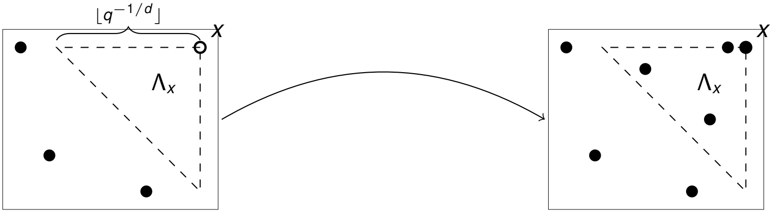


- Show that $\max_{\omega:\text{ no } \bullet \text{ in } \Lambda_x} \mathbb{P}_\omega(\tau_x < t) \to 0$ if $t = o(2^{\frac{\theta_q^2}{2d}})$ as $q \to 0$.

$\max_{\omega:\ \text{no}\ \bullet\ \text{in}\ \Lambda_x} \mathbb{P}_\omega(\tau_x < t) \to 0$ if $t = o(2^{\frac{\theta_q^2}{2d}})$

Going through a bottleneck



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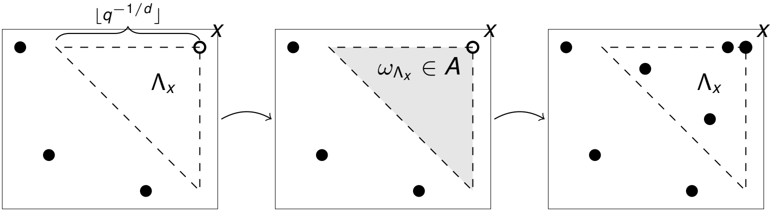

$$\max_{\omega:\text{ no } \bullet \text{ in } \Lambda_x} \mathbb{P}_\omega(\tau_x < t) \leq \max_{\omega:\text{ no } \bullet \text{ in } \Lambda_x} \mathbb{P}_\omega(\tau_A < t)$$

- CFM'16: $\exists A \in \Omega_{\Lambda_x}$ with $\mu(A) \leq 2^{-\frac{\theta_q^2}{2d}(1+o(1))}$ and $\tau_A < \tau_x$ when starting with no vacancy in Λ_x.

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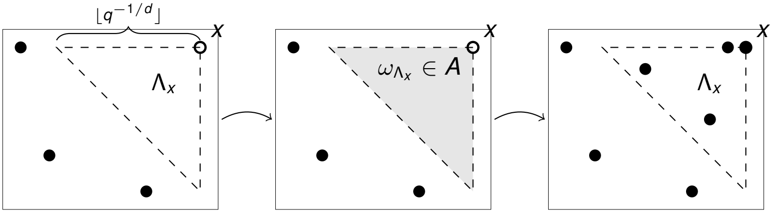

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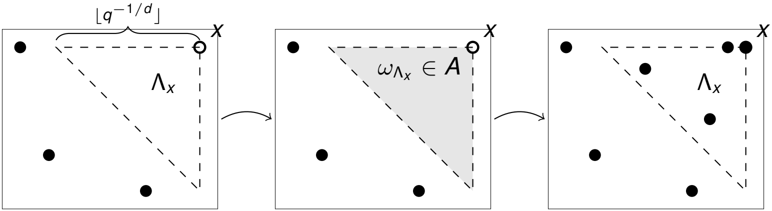

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$$\le O(t) \times 2^{-\frac{\theta_q^2}{2d}(1+o(1))}$$

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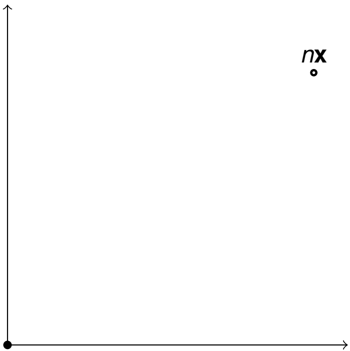

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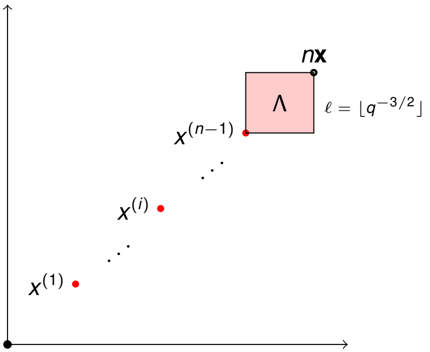

- By SMP show as $q \to 0$:

$$\max_{\omega:\, \omega_{x^{(i)}} = \bullet} \mathbb{P}_\omega(\tau_{x^{(i+1)}} > t) \to 0 \text{ if } t \gg 2^{\frac{\theta_q^2}{2d}}.$$

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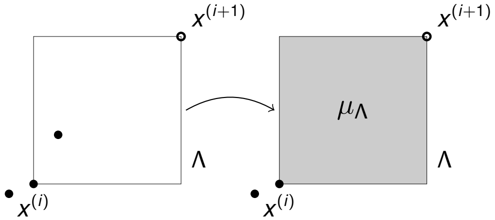

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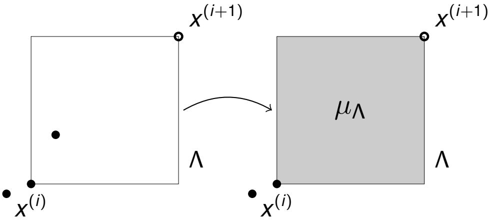

- $\mathbb{P}_\mu(\tau_{x^{(i+1)}} > t) \le e^{-t\lambda_D}$, where λ_D is the smallest λ s.t.

$$-\mathcal{L}_\Lambda f = \lambda f, \qquad f \restriction_{\{\omega :\, \omega_{x^{(i+1)}} = \bullet\}} = 0.$$

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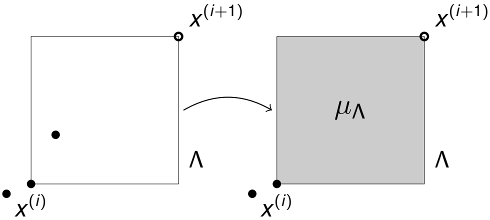

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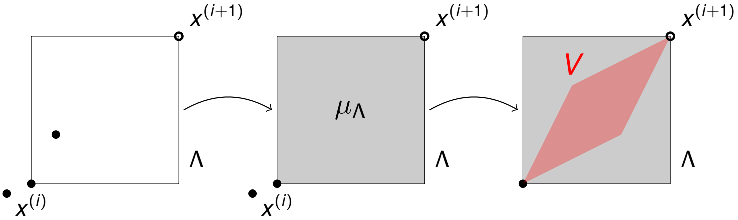
- Bad: $\lambda_D \ge q\gamma_\Lambda(q)$ but $\gamma_\Lambda(q) = \gamma_1^{(1+o(1))}(q) = 2^{-\frac{\theta_q^2}{2}(1+o(1))}$.

$\max_{\omega:\,\omega_{x^{(i)}}=\bullet} \mathbb{P}_\omega(\tau_x > t) \to 0$ if $t \gg 2^{\frac{\theta_q^2}{2d}}$



- $\mathbb{P}_\mu(\tau_{x^{(i+1)}} > t) \leq e^{-t\lambda_D}$, where λ_D is the smallest λ s.t.

$$-\mathcal{L}_\Lambda f = \lambda f, \qquad f\restriction_{\{\omega:\,\omega_{x^{(i+1)}}=\bullet\}} = 0.$$

- Better: $\lambda_D \geq q \max\{\gamma_V(q) \colon V \subset \Lambda, V \supset \{0, x^{(i+1)}\}\}$.

$$\max_{\omega:\ \omega_{x(i)}=\bullet} \mathbb{P}_\omega(\tau_x > t) \to 0 \text{ if } t \gg 2^{\frac{\theta_q^2}{2d}}$$

Proposition (Y.C., F. Martinelli '22)

For $q \to 0$ $\exists V \subset \Lambda$ containing both the lower left and top right corner s.t.

$$\gamma_V(q) \geq 2^{-\frac{\theta_q^2}{2d}(1+o(1))}.$$

$$\Rightarrow \mathbb{P}_\mu(\tau_{x^{(i+1)}} > t) \leq e^{-t2^{-\frac{\theta_q^2}{2d}(1+o(1))}}.$$

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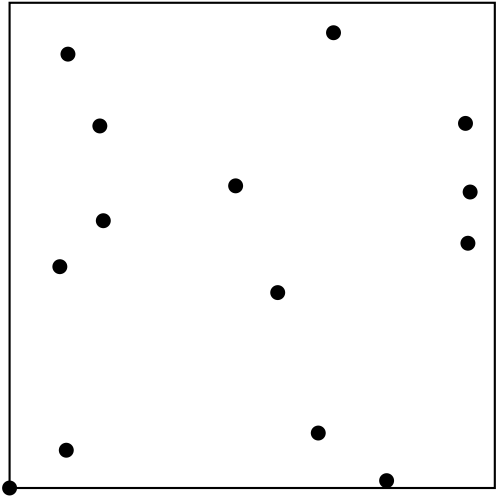
RG and Knight lattice



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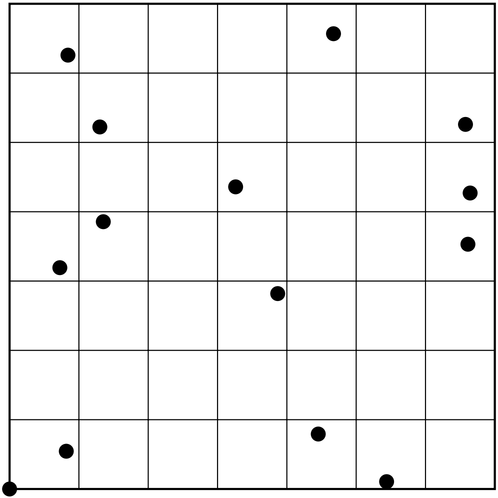

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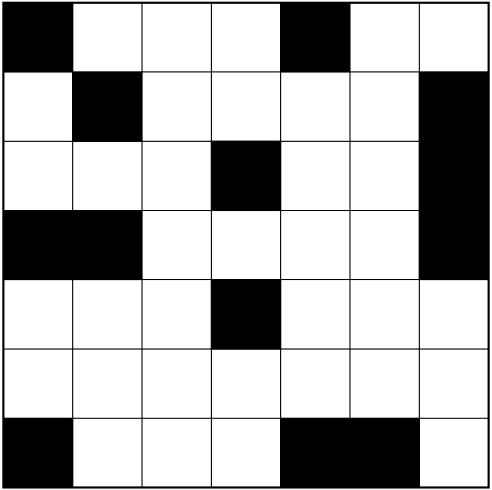

$$q_{\text{eff}} = 1 - (1-q)^{\ell^2} \gg q$$

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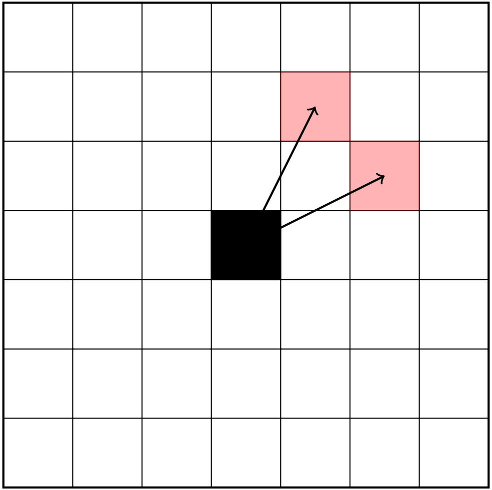

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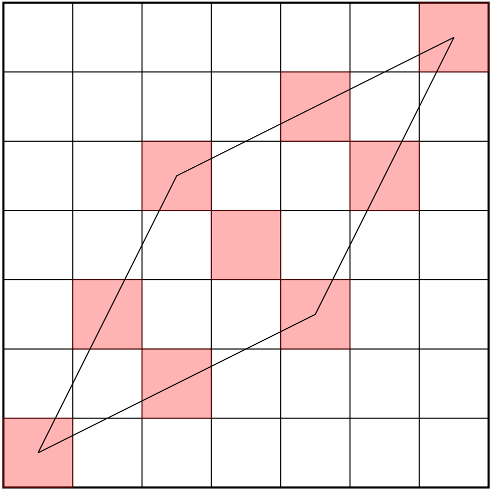

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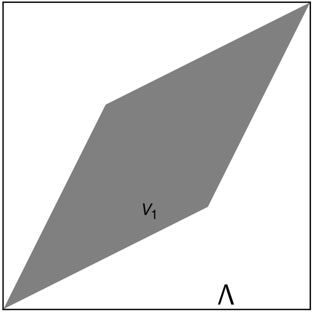

In $d = 2$:

$$\gamma_\Lambda(q) \geq 2^{-\frac{\theta_q^2}{2}(1+o(1))}$$

$$\gamma_{V_1}(q) \geq 2^{-\frac{\theta_q^2}{3}(1+o(1))}$$

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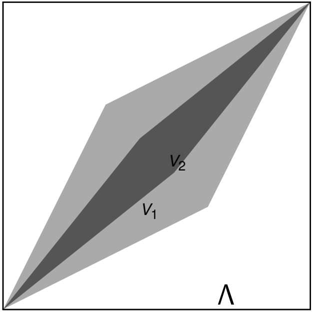

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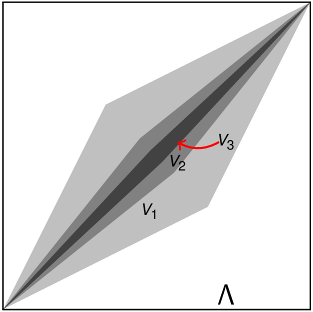
RG and Knight lattice



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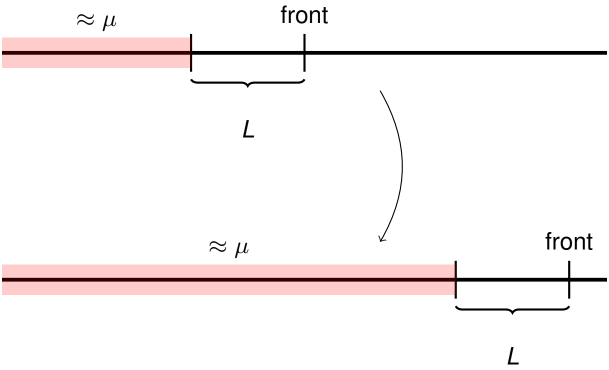
$$\gamma_{V_n}(q) \geq 2^{-\frac{\theta_q^2}{4}(1+o(1))},\ n \gg 1$$

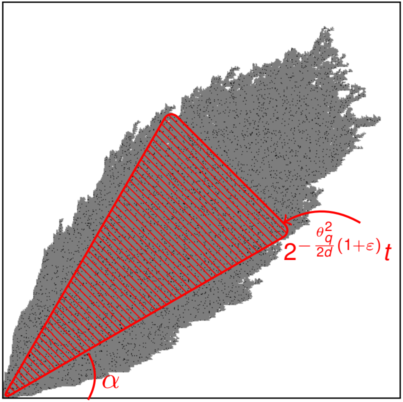
Equilibrium behind the front

Theorem (Blondel '13)

In $d = 1$, for large t the distribution at distance L behind the front approaches equilibrium exponentially in L.



Equilibrium behind front



Theorem (Y.C., F. Martinelli '22)

Vertices in red shape in equilibrium for large t and small q if $\alpha > 0$.

Cutoff

Let $\Lambda_n := \{0, \ldots, n\}^d$, $d_n(t) := \max_{\omega \in \Omega_{\Lambda_n}} \|\mathbb{P}_\omega^t - \mu_{\Lambda_n}\|_{TV}$ and consider

$$T_{\text{mix}}^{(n)}(\varepsilon) := \inf\{t > 0 \colon d_n(t) \le \varepsilon\}.$$

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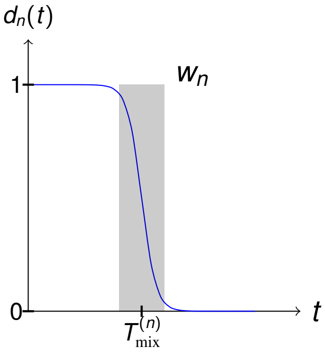

$$\lim_{\alpha \to -\infty} \liminf_{n \to \infty} d_n(T^{(n)}_{\mathrm{mix}} + \alpha w_n) = 1$$

$$\lim_{\alpha \to \infty} \liminf_{n \to \infty} d_n(T^{(n)}_{\mathrm{mix}} + \alpha w_n) = 0.$$

Cutoff

Theorem (S. Ganguly, E. Lubetzky, F. Martinelli '15)

There is a v such that the East process on $\{0, \ldots, n\}$ with parameter $0 < q < 1$ exhibits cutoff at $v^{-1}n$ with window $\sqrt{n}$.

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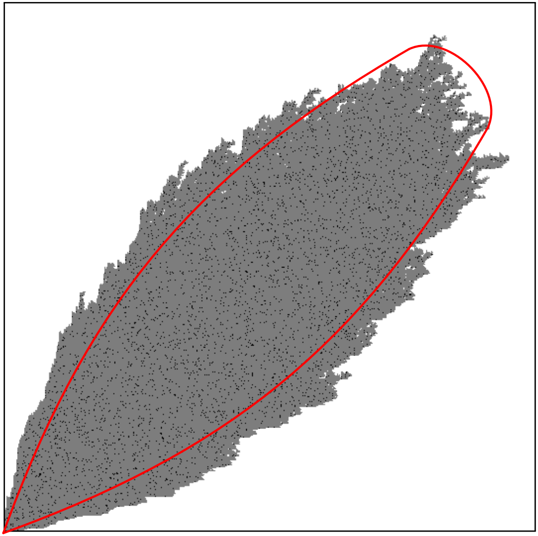
Theorem (Y.C., F. Martinelli '22)

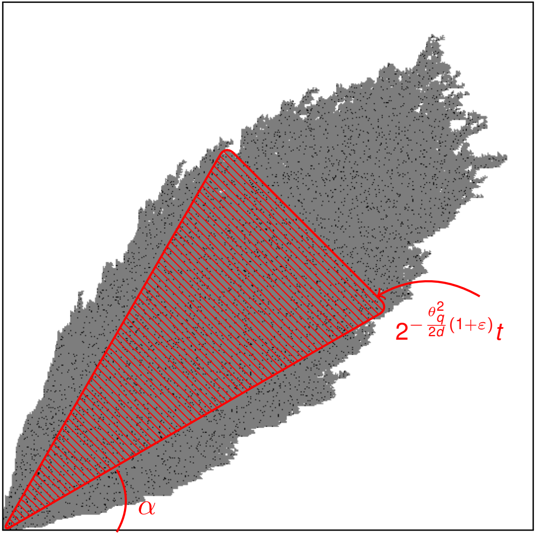
There exists $q_0 > 0$ such that the East process on $\{0, \ldots, n\}^d$ with parameter $0 < q < q_0$ exhibits cutoff at $v^{-1}n$ with window $O(n^{2/3})$.

- Because modes away from axes relax much quicker than axes modes!

Open problems



Open problems



Thank you.